METHOD OF AVERAGING THE RADIATION TRANSFER EQUATIONS FOR A NONEQUILIBRIUM PLASMA OF PLANE GEOMETRY

I. B. Kosarev

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A method is presented for averaging the radiation transfer equations for a nonequilibrium plasma of plane geometry. The averaging of transfer difference equations is carried out separately for each spectral line and for groups of a continuous spectrum. A difference scheme is obtained for one-sided radiation fluxes. Averaging coefficients are defined, characterizing absorption and emission of radiation by a cell of the difference scheme.

In solving radiation–gas dynamics problems with detailed calculation of the radiation spectrum of a gas, serious difficulty is encountered, owing to the necessity for interactive solution of the transfer equations with different values of the photon energies and the directions of their propagation. Methods of averaging the transfer equations have been found to be effective for reducing the volume of computation [1-4]. Still greater difficulty arises in the case when the plasma is nonequilibrium, and besides the equations of gas dynamics, and radiation transfer, it is necessary to solve equations of chemical kinetics. An attempt to generalize the averaging methods [1-4] to the nonequilibrium case has already been made [5]. Here, we describe a method, differing from [5] in a number of essential details, and namely in the choice of reference coefficients, more precise determination of the radiation density values determining the rate of photoionization, and, taking into consideration the dynamic Doppler shift, and the profile of a spectral line associated with macroscopic velocities in the plasma. We note that some problems connected with averaging the transfer equations applied to nonequilibrium situations have been considered in [6-8].

The proposed method of averaging the radiation transfer equations is based essentially on the emission and absorption spectra in nonequilibrium highly ionized and/or rarefied plasma, in which the distribution, as regards the degree of ionization and excited states, differs greatly from the Saha–Boltzmann, and is near to coronal [9, 10]. The emission and absorption spectra in this case are a combination of resonance spectral lines of a rather complex shape [11], far from each other (compared with their widths), on a low background continuous spectrum. Transitions between excited states cause a small contribution, almost not affecting the main picture of the spectrum. Since the mechanisms of formation of line and continuous spectra are essentially different, and also by virtue of the fact that lines situated in different regions of the energy spectrum have different effects on one or another electron shell of different ions present in the plasma, averaging in each line and in groups of the continuous spectrum is performed separately.

The averaging method described below is directly related to the Lagrange gas-dynamics scheme used in numerical computations [12], in which the density, electron concentration, ion composition, and electron and ion temperatures are identical through the volume of a cell. In the nonstationary equations determining the ion composition of the plasma, the radiation appears in the following form [9]:

$$\left(\frac{\partial N_z}{\partial t}\right)_{\mathrm{rad}} = -N_z 2\pi \sum_k \int\limits_{E_{zk}}^{\infty} \frac{\sigma_{zk}^{\mathrm{phot}}(\varepsilon)\, d\varepsilon}{\varepsilon} \int\limits_0^1 d\mu \frac{\int\limits_0^{\Delta x} I^+ dx + \int\limits_0^{\Delta x} I^- dx}{\Delta x}. \quad (1)$$

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The contribution associated with radiation in the equation for the electron energy looks similar:

$$\frac{\partial}{\partial t}\left(\frac{3}{2}N_eT_e\right)_{\text{rad}} = 2\pi\sum_z N_z\left[\sum_k \int_{E_{zk}}^{\infty} \frac{d\varepsilon\sigma_{zk}^{\text{phot}}(\varepsilon)(\varepsilon-E_{zk})}{\varepsilon}\times\right. \tag{2}$$

$$\left.\times\int_0^1 d\mu\frac{\int_0^{\Delta x} I^+dx+\int_0^{\Delta x} I^-dx}{\Delta x}+\int_{E_z^{(in)}}^{\infty} d\varepsilon\frac{\sigma_{zin}^{\text{phot}}(\varepsilon)E_{\text{Auger}}(z)}{\varepsilon}\int_0^1 d\mu\frac{\int_0^{\Delta x} I^+dx+\int_0^{\Delta x} I^-dx}{\Delta x}\right].$$

In the second term of (2) the process of autoionization decay (Auger effect) is taken into account; $\sigma_{z\,in}^{\text{phot}}(\varepsilon)$ is the photoionization cross-section of inner shells, with ionization potential $E^{(in)}$, in the dislodging of electrons from which, autoionization decay occurs; $E_{\text{Auger}}(z)$ is the disintegration energy of the escaping electron. The effect of the radiation on the ionization composition of the plasma through the rates of dielectron and photorecombination may be allowed for by introducing in the expression for the rates of the corresponding processes, the radiation density, averaged over a cell, taken in the form analogous to that used in (1), (2) [9-11]. Therefore, it may be seen that radiation enters the equation in the form of the following integrals:

$$F_1=\int_{E_{zk}}^{\infty}\frac{d\varepsilon}{\varepsilon}\sigma_{zk}^{\text{phot}}(\varepsilon)\int_0^1 d\mu\int_0^{\Delta x} I^{\pm}(\varepsilon,\ \mu)\,dx, \tag{3}$$

$$F_2=\int_{E_{zk}}^{\infty}\frac{d\varepsilon}{\varepsilon}(\varepsilon-E_{zk})\sigma_{zk}^{\text{phot}}(\varepsilon)\int_0^1 d\mu\int_0^{\Delta x} I^{\pm}(\varepsilon,\ \mu)\,dx. \tag{4}$$

1. Averaging over a Spectral Line. Owing to the sufficiently small width of a line, the quantities F_1 and F_2 may be rewritten in the form

$$F_1=\frac{\sigma_{zk}^{\text{phot}}(\varepsilon_0)}{\varepsilon_0}\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon\int_0^1 d\mu\int_0^{\Delta x} I^{\pm}dx, \tag{5}$$

$$F_2=\frac{\sigma^{\text{phot}}(\varepsilon_0)}{\varepsilon_0}(\varepsilon_0-E_{zk})\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon\int_0^1 d\mu\int_0^{\Delta x} I^{\pm}dx. \tag{6}$$

Here, ε_0 is the photon energy corresponding to the center of the line.

Let us calculate the quantity

$$\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon\int_0^1 d\mu\int_0^{\Delta x} I^{\pm}dx.$$

We write the transfer equation for the radiation intensity $I_\varepsilon^{\pm}$, in the positive (+) and negative (−) directions:

$$\mu\frac{dI_\varepsilon^{\pm}}{dx}=\pm(k_l^{\pm}+k_c)I_\varepsilon^{\pm}+(\eta_l^{\pm}+\eta_c). \tag{7}$$

Here, k_c and η_c are the absorption coefficient and radiation power in unit solid angle for a photon with energy ε in the continuous spectrum; $k_l^{\pm}$ and $\eta_l^{\pm}$ are analogous quantities for the spectral line k_c and η_c. In taking into account the dynamic Doppler shift, the quantities with the + sign differ from those with the − sign:

$$k_l^{\pm}=\frac{\pi e^2h}{mc}f_{lu}N_z\varphi^{\pm},\quad \eta_l^{\pm}=N^{(u)}A_{ul}\varphi^{\pm}\varepsilon_0/4\pi,$$

where e and m are the charge and mass of the electron; c is the velocity of light; h is Planck's constant; N_z is the number of ions z situated in the lower level. For the resonance lines considered, N_z is the number of ions situated in the ground state; $\varphi^{\pm}$ is the contour of the spectral line.

Let us introduce one-sided radiation flux

$$q_\varepsilon^\pm = 2\pi \int_0^1 \mu I_\varepsilon^\pm d\mu. \tag{8}$$

Integrating (7) with respect to μ with limits from 0 to 1:

$$\frac{\partial q_\varepsilon^\pm}{\partial x} = \mp 2\pi \int_0^1 d\mu\,(k_l^\pm + k_c)\, I_\varepsilon^\pm + 2\pi \int_0^1 d\mu\,(\eta_l^\pm + \eta_c), \tag{9}$$

and then integrating successively, first, with respect to ε in the limits $\varepsilon_0 - \Delta\varepsilon$ to $\varepsilon_0 + \Delta\varepsilon$ ($\Delta\varepsilon$ is defined by the condition that at points $\varepsilon_0 \pm \Delta\varepsilon$ the line is practically indistinguishable from the background), and, second, over the cell from 0 to Δx. For total fluxes q_1 and q_2 on the boundaries of the cell, we get

$$\pm (q_2^\pm - q_1^\pm) = \mp 2\pi \int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 d\mu \int_0^{\Delta x} dx\,(k_l^\pm + k_c)\, I_\varepsilon^\pm + 2\pi \times$$

$$\times \int_0^{\Delta x} dx \int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 d\mu\,(\eta_l^\pm + \eta_c). \tag{10}$$

In the last term of (10), taking into account normalization of the line contour, entering in the expression for $\eta_l^\pm$, integration may first be with respect to ε and then with respect to μ and x. This gives the full power of the radiation in the line, integrated over the length of a cell, i.e., the quantity determined only by the population distribution of the upper level $N^{(u)}$ over the cell and transition probability.

Let us transform the first term on the right-hand side of (10). From the mean-value theorem, we get

$$2\pi \int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 d\mu \int_0^{\Delta x} dx\,(k_l^\pm + k_c)\, I_\varepsilon^\pm = 2\pi \langle k_l^\pm + k_c \rangle \int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 d\mu \int_0^{\Delta x} dx I_\varepsilon^\pm. \tag{11}$$

This equality serves to define the mean absorption coefficient averaged over the cell and over the radiation spectrum

$$\langle k_l^\pm + k_c \rangle = \frac{\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 d\mu \int_0^{\Delta x} dx\,(k_l^\pm + k_c)\, I_\varepsilon^\pm}{\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 d\mu \int_0^{\Delta x} dx I_\varepsilon^\pm}.$$

Therefore, we may write:

$$\pm (q_2^\pm - q_1^\pm) = \mp \langle k_l^\pm + k_c \rangle\, 2\pi \int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 d\mu \int_0^{\Delta x} dx I_\varepsilon^\pm + \frac{A_{ul}\varepsilon_0}{2} \int_0^{\Delta x} N^{(u)} dx + 4\pi\eta_c \Delta x \Delta\varepsilon. \tag{12}$$

Writing out $\langle k_l^\pm + k_c \rangle$

$$\langle k_l^\pm + k_c \rangle = \frac{\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 d\mu \int_0^{\Delta x} dx \left(k_c + \frac{\pi l^2 h}{mc} f_{lu} N_z \varphi^\pm\right) I_\varepsilon^\pm}{\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 d\mu \int_0^{\Delta x} dx I_\varepsilon^\pm} = k_c + \frac{\pi l^2 h}{mc} f_{lu} N_z \langle \varphi^\pm \rangle, \tag{13}$$

i.e., the averaged line contour

$$\langle \varphi^\pm \rangle = \frac{\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 d\mu \int_0^{\Delta x} dx \varphi^\pm I_\varepsilon^\pm}{\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 d\mu \int_0^{\Delta x} dx I_\varepsilon^\pm}. \tag{14}$$

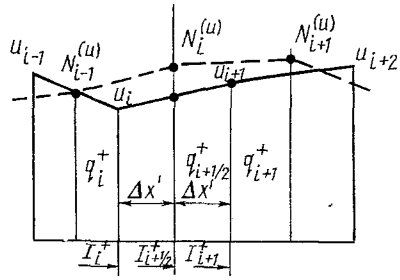

Fig. 1. Distribution of the quantities considered on a numerical grid; the radiation flux q_i, intensity I_i, and rate u_i are computed on the boundaries of the cells; $q_{i+1/2}$, $I_{i+1/2}$ and the population density of the upper level $N_i^{(u)}$ are computed at the center of the cell.

Below, the coefficient $\langle k_l^\pm + k_c \rangle$ is taken as a reference.

The population density of the upper level of line $N^{(u)}$ is determined at the center of the i-th Lagrange cell from the Biberman–Holstein integral equation [13-15]:

$$N_l N_z \langle v\sigma_{01} \rangle - N^{(u)} N_l \langle v\sigma_{10} \rangle - A_{ul} N^{(u)} + A_{ul} \int N^{(u)}(x') G(x, x') \, dx' = 0. \tag{15}$$

Here, for clarity, as the mechanism of populating the upper level, only excitation by electron collision from the ground state is taken into consideration; $\langle v\sigma_{01} \rangle$ is the rate of excitation; $\langle v\sigma_{10} \rangle$ is the rate of de-excitation of the upper state. They are connected by the relationship [10]

$$\langle v\sigma_{10} \rangle = \frac{g_0}{g_1} \langle v\sigma_{01} \rangle \exp\left(-\frac{\Delta E_{01}}{T}\right). \tag{16}$$

In the numerical solution of (15), the dependence $N^{(u)}(x)$ between the centers of the two adjacent cells in which $N_i^{(u)}$ and $N_{i+1}^{(u)}$ are computed is taken to be linear. An alternative solution of the integral equation (15), used in numerical calculations, is the determination of the radiation field in the line and the population density $N^{(u)}$ by the method of Rybicki [11, 16].

In calculations of radiation–gas dynamics problems, a solution of the Biberman–Holstein equation is made at definite moments of time for perceptible change in the macroscopic parameters of the plasma. In the intervals between these moments, the matrix of coefficients approximating the kernel G(x, x'), connecting the quantities $N^{(u)}$ at different points of the computation region, is preserved. We note that the use of approximate expressions for elements of this matrix, as, for example, in [17], gives a relatively low accuracy, since the expression used in [17] is based on an asymptotic expression for the kernel G for a uniform stationary medium.

Let us consider the calculation of the quantity $q_2^\pm - q_1^\pm$ from (10). For this we use an explicit form of the solution for the intensity $I_\varepsilon^\pm$, supposing that in the first half of the calculation cell, the rate v, appearing in the expression for the line contour, is constant and equal to $v = (1/2)[u_i + (u_i + u_{i+1}/2)] = 0.75u_i + 0.25u_{i+1}$. In the second half of the cell, $v = 0.25u_i + 0.75u_{i+1}$. Analogously with [4], we average the difference solution of the transfer equation (corresponding to the positive direction)

$$\mu \frac{dI_\varepsilon^+}{dx} = -(k_c + k_l^+) I_\varepsilon^+ + \left[(N_i^{(u)} + N^{(u)\prime} x) \frac{\varepsilon_0}{4\pi} A_{ul} \varphi^+ + \eta_c \right]. \tag{17}$$

Its solution takes the form

$$I_{i+1/2}^{+} = I_i^{+} \exp(-\tau_\varepsilon/\mu) + (S_c + S_1^{+})[1 - \exp(-\tau_\varepsilon/\mu)] - \frac{S_2^{+}\mu}{k_l^{+} + k_c}\left[1 - \frac{\tau_\varepsilon}{\mu} - \exp(-\tau_\varepsilon/\mu)\right], \tag{18}$$

where $I_{i+1/2}^{+}$ is the solution at the middle of the i-th cell (see Fig. 1);

$$\tau_\varepsilon = (k_l^{+} + k_c)\Delta x', \quad \Delta x' = \Delta x/2;$$

$$S_1^{+} = \frac{A_{ul}N_i^{(u)}\varphi^{+}}{k_l^{+} + k_c}; \quad S_c = \frac{\eta_c}{k_l^{+} + k_c}; \quad S_2^{+} = \frac{A_{ul}N^{(u)\prime}}{k_l^{+} + k_c}, \quad N^{(u)\prime} = \frac{dN^{(u)}}{dx}.$$

The derivative $dN^{(u)}/dx \mid_{-}$ from the left differs from the derivative $dN/^{(u)}/dx \mid_{+}$ from the right. Then, we have for the flux $q_{i+1/2}^{+}$

$$q_{i+1/2}^{+} = 2\pi\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 \mu I_{i+1/2} d\mu = q_i^{+}\exp(-\xi_i\langle k_l^{+} + k_c\rangle\Delta x') +$$

$$+ 2\pi\eta_c \frac{\beta_c^{+} f_c(\alpha_{ci}^{+}\langle k_l^{+} + k_c\rangle_i \Delta x')}{\langle k_l^{+} + k_c\rangle} +$$

$$+ 2\pi A_{ul}N^{(u)} \frac{\beta_1^{+} f_1^{+}(\alpha_{1i}^{+}\langle k_l^{+} + k_c\rangle_i \Delta x')}{\langle k_l^{+} + k_c\rangle_i} -$$

$$- 2\pi A_{ul}N^{(u)\prime} \frac{\beta_2^{+} f_2^{+}(\alpha_{2i}^{+}\langle k_l^{+} + k_c\rangle \Delta x')}{\langle k_l^{+} + k_c\rangle_i^2}. \tag{19}$$

Here, we introduced the following functions and parameters computed sequentially from the conditions:

a) ξ_i^{+} determined from the equation

$$2\pi\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 d\mu\mu \exp\left(-\frac{k_l^{+} + k_c}{\mu}\Delta x'\right) I_i^{+} = \exp(-\xi_i^{+}\langle k_l^{+} + k_c\rangle_i \Delta x') q_i^{+}, \tag{20}$$

b) f_c^{+}, β_c^{+}, and α_c^{+} from the equations

$$\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 \frac{\mu d\mu}{k_l^{+} + k_c} = \frac{\beta_c^{+}}{\langle k_l^{+} + k_c\rangle_i} \int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 \mu d\mu = \frac{\beta_c^{+}}{\langle k_l^{+} + k_c\rangle_i}\Delta\varepsilon,$$

$$\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 \frac{d\mu\mu \exp(-\tau_\varepsilon/\mu)}{k_l^{+} + k_c} = \frac{\beta_c^{+}}{\langle k_l^{+} + k_c\rangle_i} \int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 d\mu\mu \exp(-\tau_\varepsilon/\mu) =$$

$$= \frac{\beta_c^{+} 2\Delta\varepsilon}{\langle k_l^{+} + k_c\rangle} E_3(\alpha_c^{+}\langle k_l + k_c\rangle\Delta x'), \quad f_c(y) = \Delta\varepsilon[1 - 2E_3(y)];$$

c) f_1^{+}, β_1^{+}, and α_1^{+} from the equations

$$\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 \frac{d\mu\mu\varphi^{+}}{k_l^{+} + k_c} = \frac{\beta_1^{+}/2}{\langle k_l^{+} + k_c\rangle},$$

$$\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 \frac{d\mu\mu\varphi^{+}\exp(-\tau_\varepsilon/\mu)}{k_l^{+} + k_c} = \frac{\beta_1^{+}}{\langle k_l^{+} + k_c\rangle} E_3(\alpha_1^{+}\langle k_l^{+} + k_c\rangle\Delta x'),$$

$$f_1^{+}(y) = \frac{1}{2} - E_3(y),$$

$$\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 \frac{d\mu\mu\varphi^{+}\exp(-\tau_\varepsilon/\mu)}{k_l^{+} + k_c} = \frac{\beta_1^{+} E_3(\alpha_1^{+}\langle k_l^{+} + k_c\rangle\Delta x')}{\langle k_l^{+} + k_c\rangle};$$

d) f_2^{+}, β_2^{+}, and α_2^{+} from the equations

$$\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 \frac{d\mu\mu^2\varphi^+}{(k_l^+ + k_c)^2} = \frac{\beta_2^+/3}{\langle k_l^+ + k_c \rangle^2},$$

$$\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 \frac{d\mu\mu^2\varphi^+ [(\tau_\varepsilon/\mu) + \exp(-\tau_\varepsilon/\mu)]}{(k_l^+ + k_c)^2} =$$

$$= \frac{\beta_1^+ \frac{\Delta x'}{2}}{\langle k_l^+ + k_c} + \int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 \frac{d\mu\mu^2\varphi^+ \exp(-\tau_\varepsilon/\mu)}{(k_l^+ + k_c)^2},$$

$$\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 \frac{d\mu\mu^2\varphi^+ \exp(-\tau_\varepsilon/\mu)}{(k_l^+ + k_c)^2} = \frac{\beta_2^+}{\langle k_l^+ + k_c \rangle^2} E_4(\alpha_2^+ \langle k_l^+ + k_c \rangle_i \Delta x'),$$

$$f_2(y) = \frac{1}{3} - \frac{\beta_1^+}{\alpha_2^+ \beta_2^+} \frac{y}{2} - E_4(y).$$

This set of parameters $\xi_i{}^+$, $\beta_c{}^+$, $\alpha_c{}^+$, $\beta_1{}^+$, $\alpha_1{}^+$, $\beta_2{}^+$ and $\alpha_2{}^+$ for the quantities with + index are defined in the left half of a cell, for radiation travelling from left to right. These parameters may be defined analogously in the right half of the cell, but for radiation travelling from right to left (− index).

Interpolating and extrapolating the mass data of these parameters, and also the averaged line contour $\langle\varphi^\pm\rangle$, in correspondence with the physical characteristics of the problem, over the space−time cell in intervals between averaging, the quantities F_1 and F_2 may be calculated for each cell, by using the equations

$$\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 d\mu \int_0^{\Delta x_i} dx I_\varepsilon^+ =$$

$$= \left(q_i^+ - q_{i+1}^+ + \frac{A_{ul}}{2} \varepsilon_0 \int_0^{\Delta x_i} N^{(u)} dx + 4\pi\eta_c \Delta x_i \Delta\varepsilon \right) \Big/ 2\pi \langle k_l^+ + k_c \rangle,$$

$$\int_{\varepsilon_0-\Delta\varepsilon}^{\varepsilon_0+\Delta\varepsilon} d\varepsilon \int_0^1 d\mu \int_0^{\Delta x_i} dx I_\varepsilon^- =$$

$$= \left(q_{i+1}^- - q_i^- + \frac{A_{ul}}{2} \varepsilon_0 \int_3^{\Delta x_i} N^{(u)} dx + 4\pi\eta_c \Delta x_i \Delta\varepsilon \right) \Big/ 2\pi \langle k_l^- + k_c \rangle.$$

2. Averaging in the Continuous Spectrum. In the kinetic and power (for T_e) equations, the contribution of the continuous spectrum conforms to the equations

$$\int_{\varepsilon_1}^{\varepsilon_2} \frac{\sigma_\varepsilon}{\varepsilon} d\varepsilon \int_0^1 d\mu \int_0^{\Delta x} \frac{I_\varepsilon^\pm dx}{\Delta x} \text{ and } \int_{\varepsilon_1}^{\varepsilon_2} \frac{\sigma_\varepsilon(\varepsilon - E_{zk})}{\varepsilon} d\varepsilon \int_0^1 d\mu \int_0^{\Delta x} \frac{I_\varepsilon^\pm dx}{\Delta x}.$$

Here ε_1 and ε_2 are the lower and upper limits of the groups; ε_1 corresponds to the thresholds of photoionization of electron shells, and ε_2 is, generally speaking, infinite, but it is possible that it will be necessary in a specific situation to choose ε_2 to have a finite value, in order to then sum the contributions of different groups of quanta in the continuous spectrum, i.e., the quantity ε_2 is, for the present, not defined more precisely. Designating $H_1 = \sigma_\varepsilon/\varepsilon$ and $H_2 = \sigma_\varepsilon/\varepsilon(\varepsilon - E_{zk})$, we can write

$$\int_{\varepsilon_1}^{\varepsilon_2} H_{1,2}(\varepsilon) d\varepsilon \int_0^1 d\mu \int_0^{\Delta x} I_\varepsilon^\pm dx = \langle H_{1,2}^\pm \rangle \int_{\varepsilon_1}^{\varepsilon_2} d\varepsilon \int_0^1 d\mu \int_0^{\Delta x} I_\varepsilon^\pm dx.$$

Let us introduce the density and radiation flux in the g-th group:

$$U_g^\pm = 2\pi \int_{\varepsilon_1}^{\varepsilon_2} d\varepsilon \int_0^1 d\mu I_\varepsilon^\pm; \quad q_g^\pm = 2\pi \int_{\varepsilon_1}^{\varepsilon_2} d\varepsilon \int_0^1 \mu I_\varepsilon^\pm d\mu.$$

We write the transfer equation for the continuous spectrum:

$$\mu \frac{\partial I_\varepsilon^\pm}{\partial x} = \mp k_c I_\varepsilon^\pm + \eta_c.$$

Integrating with respect to μ, ε and over the cell, we get

$$\pm(q_{i+1}^{\pm}-q_i^{\pm})=\mp\int_{\varepsilon_1}^{\varepsilon_2}k_c d\varepsilon\int_0^{\Delta x}U_\varepsilon^{\pm}dx+2\pi\Delta x\int_{\varepsilon_1}^{\varepsilon_2}\eta_c d\varepsilon,$$

or

$$\pm(q_{i+1}^{\pm}-q_i^{\pm})=\mp\langle k^{\pm}\rangle_g\int_{\varepsilon_1}^{\varepsilon_2}d\varepsilon\int_0^{\Delta x}U_\varepsilon^{\pm}dx+2\pi\Delta x\int_{\varepsilon_1}^{\varepsilon_2}\eta_c d\varepsilon.$$

Hence, it is seen that the necessary quantity for the kinetic equations $\int_{\varepsilon_1}^{\varepsilon_2}d\varepsilon\int_0^{\Delta x}U_\varepsilon^{\pm}dx$ may be found from the equation

$$\int_{\varepsilon_1}^{\varepsilon_2}d\varepsilon\int_0^{\Delta x}U_\varepsilon^{\pm}dx=\langle k^{\pm}\rangle_g^{-1}\left(\pm(q_i^{\pm}-q_{i+1}^{\pm})+2\pi\Delta x\int_{\varepsilon_1}^{\varepsilon_2}\eta_c d\varepsilon\right)$$

and the problem, just as it did for the lines, reduces to finding $q_i^{\pm}-q_{i+1}^{\pm}$.

Let us write a more detailed expression for $\langle k^{\pm}\rangle_g$, the reference coefficient of absorption in the group:

$$\langle k_c^{\pm}\rangle_g=\frac{\int_{\varepsilon_1}^{\varepsilon_2}d\varepsilon\int_0^{\Delta x}dxU_\varepsilon^{\pm}\sum_{zk}\sigma_{zk}(\varepsilon)N_{zk}}{\int_{\varepsilon_1}^{\varepsilon_2}d\varepsilon\int_0^{\Delta x}dxU_\varepsilon^{\pm}}=$$

$$=\sum_{zk}N_{zk}\frac{\int_0^{\Delta x}dx\int_{\varepsilon_1}^{\varepsilon_2}d\varepsilon U_\varepsilon^{\pm}(x)\sigma_{zk}(\varepsilon)}{\int_0^{\Delta x}dx\int_{\varepsilon_1}^{\varepsilon_2}d\varepsilon U_\varepsilon^{\pm}(x)}=\sum_{zk}N_{zk}\langle\sigma_{zk}^{\pm}\rangle;$$

where the averaged photoabsorption cross-sections are determined from the radiation density in the g-th group:

$$\langle\sigma_{zk}^{\pm}\rangle=\frac{\int_0^{\Delta x}dx\int_{\varepsilon_1}^{\varepsilon_2}d\varepsilon U_\varepsilon^{\pm}(x)\sigma_{zk}(\varepsilon)}{\int_0^{\Delta x}dx\int_{\varepsilon_1}^{\varepsilon_2}d\varepsilon U_\varepsilon^{\pm}(x)},$$

in what follows, $\langle k_c^{\pm}\rangle_g$ is considered as the reference coefficient in the g-th group.

Let us write an equation for $q_g^{\pm}$, the flux in the group of photons of the continuous spectrum. As an example, let us take the photon flux travelling in the positive direction:

$$q_i^{+}=2\pi\int_{\varepsilon_1}^{\varepsilon_2}d\varepsilon\int_0^1 I_i^{+}\mu d\mu,\quad q_{i+1}^{+}=2\pi\int_{\varepsilon_1}^{\varepsilon_2}d\varepsilon\int_0^1 I_{i+1}^{+}\mu d\mu.$$

We write the solution of the transfer equation for the continuous spectrum in the form

$$I_{i+1}^{+}=I_i^{+}\exp\left(-\frac{k_c}{\mu}\Delta x\right)+\frac{\eta_c}{k_c}\left(1-\exp\left(-\frac{k_c}{\mu}\Delta x\right)\right).$$

Integrating with respect to μ and ε, we get

$$q_{i+1}^{+}=q_i^{+}\exp(-\xi_i^{+}\langle k_c^{+}\rangle_g\Delta x)+2\pi\int_{\varepsilon_1}^{\varepsilon_2}d\varepsilon\int_0^1\frac{\eta_c}{k_c}\mu d\mu-$$

$$-2\pi\int_{\varepsilon_1}^{\varepsilon_2}d\varepsilon\int_0^1\frac{\eta_c}{k_c}\mu\exp\left(-\frac{k_c}{\mu}\Delta x\right)d\mu.$$

Here, ξ_i^{+} is determined by the relationship

$$\int_{\varepsilon_1}^{\varepsilon_2} d\varepsilon \int_0^1 d\mu\mu I_i^+ \exp\left(-\frac{k_c}{\mu}\Delta x\right) = \left[\int_{\varepsilon_1}^{\varepsilon_2} d\varepsilon \int_0^1 d\mu\mu I_i^+\right] \exp\left(-\xi_i^+ \langle k_c^+ \rangle_g \Delta x\right),$$

$$\xi_i^+ = \frac{-\ln \dfrac{\int_{\varepsilon_1}^{\varepsilon_2} d\varepsilon \int_0^1 d\mu\mu I_i^+ \exp\left(-\frac{k_c}{\mu}\Delta x\right)}{\int_{\varepsilon_1}^{\varepsilon_2} d\varepsilon \int_0^1 d\mu\mu I_i^+}}{\langle k_c^+ \rangle_{gi} \Delta x}.$$

As in (20), it is seen that the quantity $\xi^{\pm}$ rather weakly depends on the spectrum of the radiation, which makes it possible to preserve it during a fixed number of time increments.

Let us introduce the parameter ζ_i^+ by means of the equation

$$\int_{\varepsilon_1}^{\varepsilon_2} d\varepsilon \frac{\eta_c}{k_c} \int_0^1 \mu \exp\left(-\frac{k_c}{\mu}\Delta x\right) d\mu = \left[\int_{\varepsilon_1}^{\varepsilon_2} d\varepsilon \frac{\eta_c}{k_c}\right] E_3\left(\zeta_i^+ \langle k_c^+ \rangle_{gi} \Delta x\right).$$

Then, finally, we can write:

$$q_{i+1}^+ = q_i^+ \exp\left(-\xi_i^+ \langle k_c^+ \rangle_{gi} \Delta x\right) + \left(2\pi \int_{\varepsilon_1}^{\varepsilon_2} d\varepsilon \frac{\eta_c}{k_c}\right)\left[\frac{1}{2} - E_3\left(\zeta_i^+ \langle k_c^+ \rangle_{gi} \Delta x\right)\right].$$

An analogous relation may be written for q_{i+1}^- and q_i^-.

Using the parameters $\xi_{gi}^{\pm}$ and $\zeta_{gi}^{\pm}$ for the different groups of photons, interpolating and extrapolating them over the space–time cell, it is possible to calculate from the relationship given above $\int_{\varepsilon_1}^{\varepsilon_2} d\varepsilon \int_0^{\Delta x} U_\varepsilon^{\pm} dx$, and to use them in kinetic and power equations.

Preservation and interpolation of the coefficients in the averaged transfer equations for spectral lines and groups of the continuous spectrum are carried out on the grid of the chief variables [1, 2], which are introduced in accordance with the specific physical situation. Generalization of the method to the case of spherical geometry is not difficult.

The method described makes it possible to widen the range of application of averaging methods [1-4] to the case of a nonequilibrium plasma, and to obtain a solution of one-dimensional gas dynamics problems, taking into consideration the spectral and angular composition of the radiation in considerable detail.

NOTATION

N_e, number of electrons per cm^3; N_z, number of ions of charge z per cm^3; E_{zk}, ionization potential of k-th shell of ion z; σ_{zk}^{phot}, photoionization cross-section; ε, photon energy; I^+ and I^-, intensity of radiation travelling in the positive (+) and negative (−) directions; T_e, electron temperature; Δx, size of computing cell; $\Delta x' = \Delta x/2$; μ, cosine of angle between direction of propagation of photon and the chosen axis of coordinates; k_l^+, radiation absorption coefficient in a line for radiation travelling in the positive (negative) direction; η_l^+, corresponding emission coefficient; k_c and η_c, absorption and emission coefficients in the continuous spectrum; $q^{\pm}$, one-sided radiation flux; f_{lu}, oscillator strength; A_{ul}, probability of spontaneous transition for a line; $\varphi^{\pm}$, line contour, normalized to unity; $N^{(u)}$, population density of upper state of a line; G(x, x′), kernel of integral equation; u_i, velocity of the medium, calculated at the cell boundary; v, velocity in the expression for the line contour, taking account of the dynamic Doppler shift; τ, optical thickness of the cell for quantum energy ε; $\xi^{\pm}$, $\alpha_c^{\pm}$, $\beta_c^{\pm}$, $\alpha_1^{\pm}$, $\beta_1^{\pm}$, $\alpha_2^{\pm}$, $\beta_2^{\pm}$, parameters, characterizing the absorption and emission of the computational cell for a spectral line, which go into the averaged difference equations for one-sided fluxes; f_c, f_1, f_2, input functions to those equations; $\xi_g^{\pm}$, $\zeta_g^{\pm}$, parameters characterizing the absorption and emission of the continuous spectrum; $E_n(x) = \int_1^{\infty} \frac{\exp(-xt)}{t_n} dt$, exponential integral.

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